

Bipartite Networks (Advanced Concepts)

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Recap: Bipartite Networks

- Two distinct node sets

- Links only occur between sets

- Also called affiliation or two-mode networks
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Bi-Adjacency Matrix Representation

Rows represent one node set

Columns represent the other node set

Entries indicate whether a connection exists

Example Matrix

Students × Courses

A: Math=1, Physics=0

B: Math=0, Physics=1

C: Math=1, Physics=0

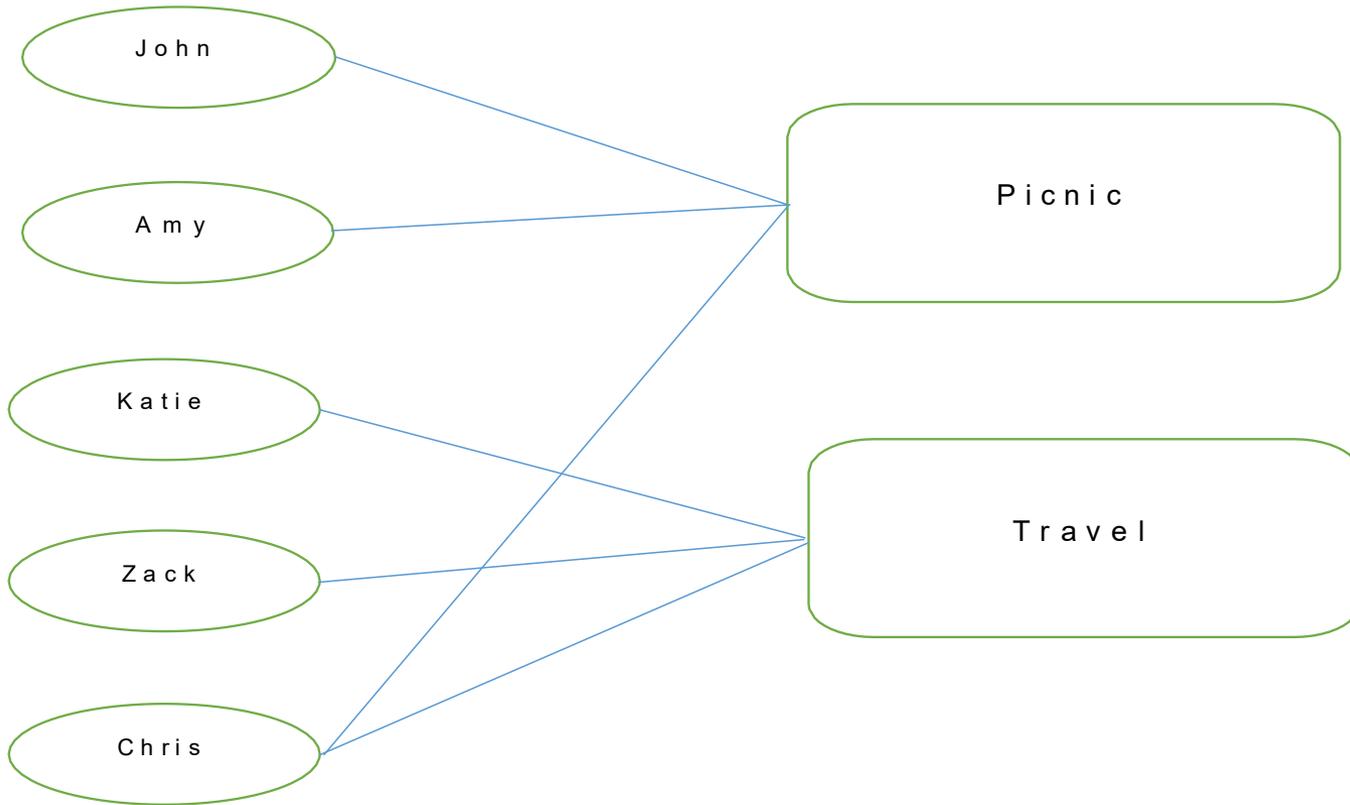
Matrix multiplication

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} b_{11}a_{11} + b_{12}a_{21} + b_{13}a_{31} & b_{11}a_{12} + b_{12}a_{22} + b_{13}a_{32} \\ b_{21}a_{11} + b_{22}a_{21} + b_{23}a_{31} & b_{21}a_{12} + b_{22}a_{22} + b_{23}a_{32} \end{pmatrix}$$

The following artificial dataset



Converting bipartite to one- mode network

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix}$$

Actor by event matrix multiplies with event by actor matrix =
actor by actor matrix.

Event by actor multiply by actor by event

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

Event by actor matrix multiplies with actor by event matrix =
event by event matrix

The result

	John	Amy	Katie	Zach	Chris	Picnic	Travel
John	1	1	0	0	1	1	0
Amy	1	1	0	0	1	1	0
Katie	0	0	1	1	1	0	1
Zach	0	0	1	1	1	0	1
Chris	1	1	1	1	2	1	1
Picnic	1	1	0	0	1	3	1
Travel	0	0	1	1	1	1	3

Student Projection Example

- A and C take Math → connection
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- B shares no course with others
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- Result: Student collaboration network

Weighting in Projections

Edge weights can represent shared connections

Example: number of shared courses

Helps measure strength of relationships

Applications in Research



Scientific collaboration networks



Recommendation systems



Event participation networks



Worker/CEO organizational linkage

Software Tools

- Gephi

- NetworkX (Python)

- igraph (R/Python)

- UCINET

Conclusion

Bipartite networks model affiliation relationships

Projection enables further analysis

Important concept in modern network science