



# Social Network Analysis: Betweenness Centrality

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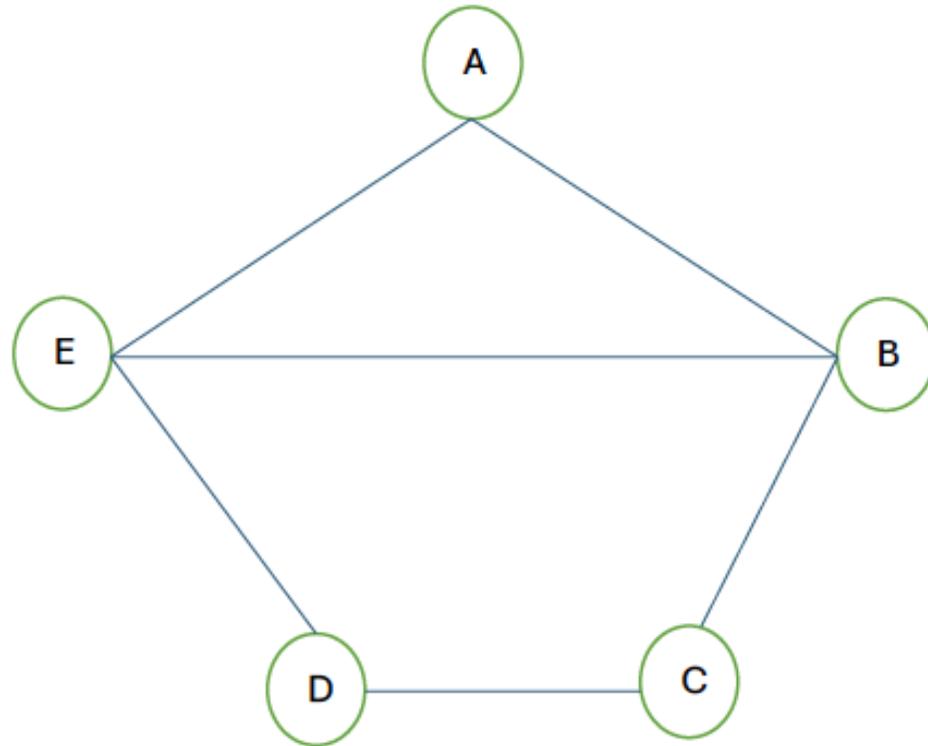
# Formula

- For a node  $v$ :
- $C_B(v) = \sum_{s \neq v \neq t} \sigma_{st}(v)$
- Where:
- $s, t =$  all pairs of distinct nodes
- $\sigma_{st}(v) =$  number of those shortest paths from  $s$  to  $t$  that pass through  $v$

# Normalizing betweenness centrality

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- For directed graph, the max number of chances of sitting on shortest paths for all pairs of any given node in a graph is
  - $P_{N-1}^2 = \frac{(N-1)!}{(N-3)!} = (N-2) \times (N-1)$  *directed graphs*
  - $C_{N-1}^2 = \frac{(N-1)!}{(N-3)!/2} = \frac{(N-2)(N-1)}{2}$  *undirected graphs*
  - Normalizing the betweenness centrality would be
  - $C'_B(v) = \frac{\sigma_{st}(v)}{(N-2)(N-1)}$
  - For undirected graphs, normalizing the betweenness centrality would be
  - $C'_B(v) = \frac{\sigma_{st}(v)}{(N-2)(N-1)/2}$

# Calculating the betweenness centrality for the following graph



# Empirical example

Node	Betweenness Centrality	paths
A	0	BC, BD, BE, CD, CE, DE
B	2	AC, AD, AE, CD, CE, DE
C	1	AB, AD, AE, BD, BE, DE
D	1	AB, AC, AE, BC, BE, CE
E	2	AB, AC, AE, BC, BD, CD